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SWs and DLs in IA solitary waves in e-p-i degenerate dense plasma

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The nonlinear propagation of ion-acoustic waves in an unmagnetized collisionless degenerate dense plasma (containing degenerate electron, positron, and ion fluids) has been theoretically investigated. This fluid model, which is valid for both the non-relativistic and ultra-relativistic limits has been employed with the reductive perturbation method. The standard Gardner (sG) equation has been derived, and numerically examined. The dynamics of electrons, positrons and ions on the IA (ion-acoustic) solitary waves (SWs) and double layers (DLs) that are found to exit in a degenerate dense plasma by taking the effect of different plasma parameters in the plasma fluid into account. The relevance of our results in astrophysical objects like white dwarfs and neutron stars, which are of scientific interest, are briefly discussed.

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I. INTRODUCTION

The propagation of the ion-acoustic waves are very important from both the academic point of view and from the view of its vital role in understanding the electrostatic disturbances in space and laboratory plasma. The physics of quantum plasmas, rapidly grown beyond conventional plasmas found in space or laboratory for many years [1, 2]. This is mainly due to the potential applications of quantum plasmas in different areas of scientific and technological importance [3–6]. It is a common idea that electron-positron plasmas have presumably appeared in the early universe [7, 8] and are frequently encountered in active galactic nuclei [9] and in pulsar magnetospheres [10, 11]. This electron-positron plasma is usually characterized as a fully ionized gas consisting of electrons and positrons of equal masses. Recently, there has been a great deal of interest in studying linear as well as nonlinear wave motions in such plasmas [12, 13]. The nonlinear studies have been focused on the nonlinear self-consistent structures [12–14] such as envelope solitons, vortices, etc. However, most of the astrophysical plasmas usually contains ions, in addition to the electrons and positrons. Clearly, the properties of wave motions in an electron-positron-ion plasma should be different from those in two-component electron-positron plasmas. For example, Rizzato [15] and Berezhiani *et al.* [16] have investigated envelope solitons of electromagnetic waves in three-component electron-positron-ion plasmas.

The electron-positron plasmas are thought to be generated naturally by pair production in high energy processes in the vicinity of several astrophysical objects as well as produced in laboratory plasmas experiments with a finite life time [17]. Because of the long life time of the positrons, most of the astrophysical [18] and laboratory plasmas become an admixture of electrons, positrons, and ions. It has also been shown that over a wide range of parameters, annihilation of electrons and positrons, which is the analog of recombination in plasma composed of ions and electrons, is relatively unimportant in classical, [19] as well as in dense quantum plasmas [20] to study the collective plasma oscillations. The ultradense degenerate electron positron plasmas with ions are believed to be found in compact astrophysical bodies like neutron stars and the inner layers of white dwarfs [20–23] as well as in intense laser-matter interaction experiments [24, 25]. Therefore, it seems important to study the influence of quantum effects on dense e-p-i plasmas. Several authors have theoretically investigated the collective effects in dense unmagnetized and magnetized e-p-i quantum plasmas under the assumption of low-phase velocity (in comparison with electron/positron Fermi velocity) [26–28]. In these studies, the authors have focused on the lower order quantum corrections appearing in the well known classical modes.

A dense plasma is usually characterized as cold and degenerate such as that encountered in metals and semiconductors. However, it has been remarked that a hot fusion plasma such as that found in dense steller objects (e.g., white dwarfs) may also be considered as quantum degenerate plsma [1]. In such environments the production of positrons is and a degenerate plasma of electron-positron-ion can be expected. The main objection to the existence of dense electron-positron-ion plasma may be high electron-positron annihilation rate which is naturally expected where the electron and positron density are very high. In a typical white dwarf star the electron density can be as high as $10^{28}cm^{-3}$, however, for massive stars [29] such as that for a collapsing white dwarf, this value can even be much higher [23]. The propagation and collision of small-amplitude ion-acoustic waves in ultra relativistic plasma have been already investigated [30, 31].

Now-a-days, a number of authors have become interested to study the properties of matter under extreme conditions [32–35]. Recently, a number of theoretical investigations have also been made of the nonlinear propagation of electrostatic waves in degenerate quantum plasma by a number of authors [54–56] etc. However, these investigations are based on the electron equation

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of state valid for the non-relativistic limit. Some investigations have been made of the nonlinear propagation of electrostatic waves in a degenerate dense plasma based on the degenerate electron equation of state valid for ultra-relativistic limit [36–38].We are interested to study the dissipasion relation of the ion-acoustic waves in a degenerate e-p-i plasma system where we added positrons for the rather long lifetime of positrons, most of the astrophysical [9, 11–13, 18, 23, 39, 40] as we have mentioned in the introductory chapter. The pressure for ion fluid can be given by the following equation

$$P_i = K_i n_i^{\alpha}, \tag{1}$$

where

$$\alpha = \frac{5}{3}; \quad K_i = \frac{3}{5}\left(\frac{\pi}{3}\right)^{\frac{1}{3}} \frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c \hbar c, \tag{2}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}\ cm$, and $\hbar$ is the Planck constant divided by 2π). While for the electron fluid,

$$P_e = K_e n_e^{\gamma}, \tag{3}$$

where

$$\gamma = \alpha; K_e = K_i \text{ for nonrelativistic limit, and} \tag{4}$$

$$\gamma = \frac{4}{3}; \quad K_e = \frac{3}{4}\left(\frac{\pi^2}{9}\right)^{\frac{1}{3}} \hbar c \simeq \frac{3}{4}\hbar c, \tag{5}$$

in the ultra-relativistic limit [32–34, 36, 38].

Therefore, in our present investigation, we consider a degenerate dense plasma system in absence of the magnetic field or heavy dust grains, but containing non-relativistic degenerate cold ion fluid, both non-relativistic and ultra-relativistic degenerate electrons and positrons fluid where the ion is the heavier element among all other elements. The model is relevant to compact interstellar objects (e. g., white dwarf, neutron star, etc.). Recently, many authors [1, 36, 38, 43–52], etc. have used the pressure laws (3) to (5) investigate the linear and nonlinear properties of electrostatic and electromagnetic waves, by using the non-relativistic quantum hydrodynamic (QHD) [1] and quantum-magnetohydrodynamic(Q-MHD) [45] models and by assuming either immobile ions or non-degenerate uncorrelated mobile ions. Again in this present days, some authors [54–56] has made a number of theoretical investigations on the nonlinear propagation of electrostatic waves in degenerate quantum plasma. Still now, there is no theoretical investigation has been made to study the extreme condition of matter for both non-relativistic and ultra-relativistic limits on the propagation of electrostatic solitary waves (SWs) and double layers (DLs) in a degenerate dense plasma system. Therefore, in our paper we have studied the properties of the SWs and DLs considering a degenerate dense plasma containing degenerate electron-ion fluid (both non-relativistic and ultra-relativistic limits) with the degenerate positron to study the basic features of the electrostatic soliton and double layer structures with the solutions of standard Gardner equation. Our considered model is relevant to compact interstellar objects (i.e. white dwarf, neutron star, black hole, etc.).

II. GOVERNING EQUATIONS

We consider an unmagnetized collisionless three component degenerate dense plasma system consisting of non-relativistic degenerate cold degenerate ion fluid and both non-relativistic and ultra-relativistic degenerate electrons and positrons fluids. We assume that the ion is the heavier element among all other considering elements. The dynamics of the one dimensional ion-acoustic waves in such a three component degenerate dense plasma system is governed by

$$\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s u_s) = 0, \tag{6}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} + \frac{K_1}{n_i}\frac{\partial n_i^{\alpha}}{\partial x} - \eta \frac{\partial^2 u_i}{\partial x^2} = 0, \tag{7}$$

$$n_e \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_e^{\gamma}}{\partial x} = 0, \tag{8}$$

$$n_p \frac{\partial \phi}{\partial x} - K_2 \frac{\partial n_p^{\gamma}}{\partial x} = 0, \tag{9}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e \alpha_e - n_i - \alpha_p n_p, \tag{10}$$

where n_s is the plasma number density of the species s (s = e for electron, i for ion, and p for positron) normalized by its equilibrium value n_{so} (n_{e0}), u_s is the plasma species fluid speed normalized by $C_{im} = (m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the electron (ion) rest mass mass and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $m_e c^2/e$ with e being the magnitude of the charge of an electron, the time variable (t) is normalized by $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$, and the space variable (x) is normalized by $\lambda_m = (m_e c^2/4\pi n_0 e^2)^{1/2}$. The coefficient of viscosity η is a normalized quantity given by $\omega_i \lambda_{mi}^2 m_s n_{s0}$, and α_e is the ratio of the number density of electron and ion (n_e/n_i) and $\alpha_p$is the ratio of the number density of positron and ion (n_p/n_i). The constants $K_1 = n_0^{\alpha-1} K_i/{m_i}^2 {C_i}^2$ and $K_2 = n_0^{\gamma-1} K_e/m_i {C_i}^2$=$n_0^{\gamma-1} K_p/m_i {C_i}^2$.

III. DERIVATION OF K-DV EQUATION

Now we derive a dynamical equation for the nonlinear propagation of the ion-acoustic solitary waves by using (6 - 10). To do so, we employ a reductive perturbation technique to examine electrostatic perturbations propagating in the relativistic degenerate dense plasma due to the effect of dissipation, we first introduce the stretched coordinates [57]

$$\zeta = \epsilon^{1/2}(x - V_p t), \tag{11}$$
$$\tau = \epsilon^{3/2} t, \tag{12}$$

where V_p is the wave phase speed (ω/k with ω being angular frequency and k being the wave number of the perturbation mode), and ϵ is a smallness parameter measuring the weakness of the dispersion ($0 < \epsilon < 1$). We then expand n_i, n_e, u_i, and ϕ, in power series of ϵ:

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots, \tag{13}$$
$$n_e = 1 + \epsilon n_e^{(1)} + \epsilon^2 n_e^{(2)} + \cdots, \tag{14}$$
$$n_p = 1 + \epsilon n_p^{(1)} + \epsilon^2 n_p^{(2)} + \cdots, \tag{15}$$
$$u_i = \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots, \tag{16}$$
$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots, \tag{17}$$
$$\rho = \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots, \tag{18}$$
$$\tag{19}$$

and develop equations in various powers of ϵ. To the lowest order in ϵ, using equations (11)-(17) into equations (6) - (10) we get as, $u_i^{(1)} = V_p \phi^{(1)}/(V_p^2 - K_1')$, $n_i^{(1)} = \phi^{(1)}/(V_p^2 - K_1')$, $n_e^{(1)} = n_p^{(1)} = \phi^{(1)}/K_2'$, and $V_p = \sqrt{(\frac{K_2'}{\alpha_e - \alpha_p} + K_1')}$, where $K_1 = n_0^{\alpha-1} K_i / m_i {C_i}^2$ and $K_2 = n_0^{\gamma-1} K_e / m_i {C_i}^2 = n_0^{\gamma-1} K_p / m_i {C_i}^2$. The relation $V_p = \sqrt{(\frac{K_2'}{\alpha_e - \alpha_p} + K_1')}$ represents the dispersion relation for the ion-acoustic type electrostatic waves in the degenerate plasma under consideration.

We are interested in studying the nonlinear propagation of these dissipative ion-acoustic type electrostatic waves in a three components degenerate plasma. To the next higher order in ϵ, we obtain a set of equations

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(2)}}{\partial \zeta} - \frac{\partial}{\partial \zeta}[u_s^{(2)} + n_s^{(1)} u_s^{(1)}] = 0, \tag{20}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(2)}}{\partial \zeta} + u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \zeta} + \frac{\partial \phi^{(2)}}{\partial \zeta} + K_1' \frac{\partial}{\partial \zeta}\left[n_i^{(2)} + \frac{(\alpha-2)}{2}(n_i^{(1)})^2\right] = 0, \tag{21}$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_e^{(2)} + \frac{(\gamma-2)}{2}(n_e^{(1)})^2\right] = 0, \tag{22}$$

$$\frac{\partial \phi^{(2)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_p^{(2)} + \frac{(\gamma-2)}{2}(n_p^{(1)})^2\right] = 0, \tag{23}$$

$$0 = \alpha_e n_e^{(2)} - n_i^{(2)} - \alpha_p n_p^{(2)}. \tag{24}$$

Now, combining (20-24) we deduce a K-dV equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + B \frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \tag{25}$$

where the value of A and B are given by

$$A = \frac{(V_p^2 - K_1')^2}{2V_p}\left[\frac{3V_p^2 + K_1'(\alpha-2)}{(V_p^2 - K_1')^3} + \frac{(\gamma-2)(\alpha_e - \alpha_p)}{{K_2'}^2}\right], \tag{26}$$

$$B = \frac{(V_p^2 - K_1')^2}{2V_p}. \tag{27}$$

The solitary wave solution of (25) is

$$\phi^{(1)} = \phi_m \text{sech}^2\left(\frac{\xi}{\delta}\right), \tag{28}$$

where the special coordinate, $\xi = \zeta - u_0 \tau$, the amplitude, $\phi_m = 3u_0/A$, and the width, $\Delta = (4B/u_0)^{1/2}$.

IV. DERIVATION OF MODIFIED K-DV EQUATION

The K-dV equation is the result of the second order calculation of the ϵ. From the third order calculation, which utilizes another set of stretched coordinate, a modified k-dV (mk-dV) equation is obtained to describe the nonlinear evolution near this critical parameter. The stretched coordinates for mk-dV equation is

$$\zeta = \epsilon(x - V_p t), \tag{29}$$
$$\tau = \epsilon^3 t, \tag{30}$$

By using and , we find the same values of $u_i^{(1)}$, $n_i^{(1)}$, $n_e^{(1)}$, $n_p^{(1)}$, and V_p, as like as that of the k-dV. To the next higher order of ϵ, we obtain a set of equations, which after using the values of $u_i^{(1)}$, $n_i^{(1)}$, $n_e^{(1)}$, $n_p^{(1)}$, and V_p,can be simplified as

$$n_i^{(2)} = \frac{3V_p^2 + K_1'(\alpha-2)}{2(V_p^2 - K_1')^3}(\phi^{(1)})^2 + \frac{\phi^{(2)}}{V_p^2 - K_1'}, \tag{31}$$

$$u_i^{(2)} = \frac{V_p K_1'}{(V_p^2 - K_1')^3}(\phi^{(1)})^2 + \frac{V_p^3 + V_p K_1'(\alpha-2)}{2(V_p^2 - K_1')^3}(\phi^{(1)})^2 + \frac{V_p \phi^{(2)}}{V_p^2 - K_1'}, \tag{32}$$

$$n_e^{(2)} = \frac{1}{K_2'}\phi^{(2)} - \frac{\gamma-2}{2(K_2')^2}(\phi^{(1)})^2, \tag{33}$$

$$n_p^{(2)} = \frac{1}{K_2'}\phi^{(2)} - \frac{\gamma-2}{2(K_2')^2}(\phi^{(1)})^2, \tag{34}$$

$$\rho^{(2)} = \frac{1}{2}A(\phi^{(1)})^2, \tag{35}$$

$$A = \frac{3V_p^2 + K_1'(\alpha-2)}{(V_p^2 - K_1')^3} - \frac{(\gamma-2)(\alpha_p - \alpha_e)}{(K_2')^2}, \tag{36}$$

To next higher order in ϵ, we obtain a set of equations:

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[u_s^{(3)} + n_s^{(1)} u_s^{(2)} + n_s^{(2)} u_s^{(1)}] = 0, \tag{37}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(3)}}{\partial \zeta} + \frac{\partial u_i^{(1)} u_i^{(2)}}{\partial \zeta} + \frac{\partial \phi^{(3)}}{\partial \zeta} + K_1' \frac{\partial}{\partial \zeta}\left[n_i^{(3)} + (\alpha-2)(n_i^{(1)} n_i^{(2)})\right] + \frac{(\alpha-2)(\alpha-3)}{2}(n_i^{(1)})^2 \frac{\partial n_i^{(1)}}{\partial \zeta} = 0, \quad (38)$$

$$\frac{\partial \phi^{(3)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_e^{(3)} + (\gamma-2)(n_e^{(1)} n_e^{(2)}) + \frac{(\gamma-2)(\gamma-3)}{6}(n_e^{(1)})^3\right] = 0, \quad (39)$$

$$\frac{\partial \phi^{(3)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}\left[n_p^{(3)} + (\gamma-2)(n_p^{(1)} n_p^{(2)}) + \frac{(\gamma-2)(\gamma-3)}{6}(n_p^{(1)})^3\right] = 0, \quad (40)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} - \alpha_e n_e^{(3)} + n_i^{(3)} + \alpha_p n_p^{(3)} = 0. \quad (41)$$

Now combining (37-41), and using the values of $n_i^{(1)}$, $n_i^{(2)}$, $u_i^{(1)}$, $u_i^{(2)}$, $n_e^{(1)}$, $n_e^{(2)}$, $n_p^{(1)}$, $n_p^{(2)}$ and ρ^2, we obtain of the form:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + ab{\phi^{(1)}}^2 \frac{\partial \phi^{(1)}}{\partial \zeta} + b\frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0, \quad (42)$$

where

$$a = \left[\frac{15V_p^4 + 12V_p^2 K_1' + 18V_p^2 K_1'(\alpha-2) + 3(K_1')^2(\alpha-2)^2}{2(V_p^2 - K_1')^5} + \frac{K_1'(\alpha-2)(\alpha-3)}{2(V_p^2-K_1')^4} + \frac{(2\gamma^2 - 7\gamma + 6)(\alpha_p - \alpha_e)}{2{K_2'}^3}\right], \quad (43)$$

$$b = \frac{(V_p^2 - K_1')^2}{2V_p}. \quad (44)$$

Equation (42) is known as mK-dV equation. The stationary localized solution of (42) is, therefore, directly given by

$$\phi^{(1)} = \phi_m \mathrm{sech}\left(\frac{\xi}{\Delta}\right), \quad (45)$$

where the amplitude ϕ_m and the width Δ are given by $\phi_m = \sqrt{\frac{6U_0}{ab}}$ and $\Delta = \frac{1}{\phi_m \sqrt{\gamma}}$, $\gamma = \frac{a}{6}$.

V. DERIVATION OF STANDARD GARDNER EQUATION

It is obvious from (37) that $A = 0$ since $\phi^{(1)} = 0$. One can find that $A = 0$ at its critical value $\alpha_e = (\alpha_e)_c$ (which is a solution of $A = 0$). So, for α_e around its critical value $(\alpha_e)_c = \frac{-3K_2'}{2(K_1' + \alpha(K_1'))} - \frac{\sqrt{8\Gamma + 8\alpha\Gamma - 4\gamma\Gamma - 4\alpha\gamma\Gamma + 9{K_2'}^2}}{2(K_1') + \alpha(K_1')} + \frac{2(K_1')(\alpha_p) + 2\alpha(K_1')(\alpha_p)}{2(K_1' + \alpha(K_1'))}$ where $\Gamma = (K_1')(K_2')$, $A = A_0$ can be expressed as

$$A_0 = \left(\frac{\partial A}{\partial \alpha_e}\right)_{\alpha_e = (\alpha_e)_c} |\alpha_e - (\alpha_e)_c| = sA_\alpha \epsilon, \quad (46)$$

where $|\alpha_e - (\alpha_e)_c|$ is a dimensionless parameter, and can be taken as the expansion parameter ϵ, i.e. $|\alpha_e - (\alpha_e)_c| = \epsilon$, and where

$$A_\alpha = \frac{K_2'(-2 + \gamma + 6\alpha_e - 6\alpha_p)}{(K_2')^3} + \frac{3(1+\alpha)K_1'(\alpha_e - \alpha_p)^2}{(K_2')^3} \quad (47)$$

and $s = 1$ for $\alpha_e > (\alpha_e)_c$ and $s = -1$ for $\alpha_e < (\alpha_e)_c$. So, for $\alpha_e = (\alpha_e)_c$, we can express $\rho^{(2)}$ as

$$\rho^{(2)} \simeq \frac{1}{2} s\epsilon A_\alpha {\phi^{(1)}}^2 \quad (48)$$

This means that for $\alpha_e \neq (\alpha_e)_c$, $\rho^{(2)}$ must be included in the third order Poisson's equation. To the next higher order in ϵ, we obtain the third set of equations:

$$\frac{\partial n_s^{(1)}}{\partial \tau} - V_p \frac{\partial n_s^{(3)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[u_s^{(3)} + n_s^{(1)} u_s^{(2)} + n_s^{(2)} u_s^{(1)}] = 0, \quad (49)$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V_p \frac{\partial u_i^{(3)}}{\partial \zeta} + \frac{\partial u_i^{(1)} u_i^{(2)}}{\partial \zeta} + \frac{\partial \phi^{(3)}}{\partial \zeta} + K_1' \frac{\partial}{\partial \zeta}\left[n_i^{(3)} + (\alpha-2)(n_i^{(1)} n_i^{(2)})\right] + \frac{(\alpha-2)(\alpha-3)}{2}(n_i^{(1)})^2 \frac{\partial n_i^{(1)}}{\partial \zeta} = 0, \quad (50)$$

$$\frac{\partial \phi^{(3)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}[n_e^{(3)} + (\gamma-2)(n_e^{(1)} n_e^{(2)}) + \frac{(\gamma-2)(\gamma-3)}{6}(n_e^{(1)})^3] = 0, \quad (51)$$

$$\frac{\partial \phi^{(3)}}{\partial \zeta} - K_2' \frac{\partial}{\partial \zeta}[n_p^{(3)} + (\gamma-2)(n_p^{(1)} n_p^{(2)}) + \frac{(\gamma-2)(\gamma-3)}{6}(n_p^{(1)})^3] = 0, \quad (52)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \zeta^2} + \frac{1}{2} sA_\alpha {\phi^{(1)}}^2 + \rho^{(3)} = 0. \quad (53)$$

where $\rho^{(3)} = -\alpha_e n_e^{(3)} + n_i^{(3)} + \alpha_p n_p^{(3)}$. Now,combining equations (31)-(35) and (48)-(53), we obtain a equation of the form:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + bsA_\mu \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \zeta} + ab{\phi^{(1)}}^2 \frac{\partial \phi^{(1)}}{\partial \zeta} + b\frac{\partial^3 \phi^{(1)}}{\partial \zeta^3} = 0. \quad (54)$$

where a and b are same as before. Equation (54) is known as standard Gardner (sG) equation. It is often called mixed mK-dV (mmK-dV) equation, because it contains both ${\phi^{(1)}}^2$ term of K-dV and ${\phi^{(1)}}^3$ term of mk-dV. Equation (54) is valid for (α_e) near its critical value $(\alpha_e)_c$. As

(54) contains both $\phi^{(1)}$ and ${\phi^{(1)}}^2$ terms, it supports both the SWs and DLs solution. It is important to note that if we neglect the ${\phi^{(1)}}^2$ term, this equation reduces to mK-dV equation, and to K-dV equation by using a lower order stretching viz. $\zeta = \epsilon^{1/2}(x - V_p t)$, and $\tau = \epsilon^3 t$.

The exact analytical solution of (54) is not possible. Therefore, we have numerically solved (54), and have studied the effects of planar geometry IA GSs and DLs. The stationary SW and SDL solution of the sG equation [i.e. (54)] is obtained by considering a moving frame (moving with speed U_0) $\xi = \zeta - U_0\tau$, and imposing all the appropriate boundary conditions for the SW and DL solution, including $\phi^{(1)} \to 0$, $d\phi^{(1)}/d\xi \to 0$, $d^2\phi^{(1)}/d\xi^2 \to 0$ at $\xi \to -\infty$.These boundary conditions allow us to have two solutions to express the sG equation [i.e. (54)], as one is the stationary SW solution and another is DL solution. The stationary SW solution of sG equation [i.e. (54)] can be written as

$$\phi^{(1)} = \left[\frac{1}{\phi_{m2}} - \left(\frac{1}{\phi_{m2}} - \frac{1}{\phi_{m1}}\right)\cosh^2\left(\frac{\xi}{\delta}\right)\right]^{-1}, \quad (55)$$

where δ is the width of the SWs. $\phi_{m1,2}$ and δ are given by

$$\phi_{m1,2} = \phi_m[1 \mp \sqrt{1 + \frac{U_0}{V_0}}] \quad (56)$$

$$V_0 = \frac{s^2 B}{6\alpha}, \quad (57)$$

$$U_0 = \frac{sB}{3}\phi_{m1,2} + \frac{\beta}{6}{\phi_{m1,2}}^2 \quad (58)$$

$$\delta = \frac{2}{\sqrt{-\gamma\phi_{m1}\phi_{m2}}} \quad (59)$$

$$\gamma = \frac{\alpha}{6} \quad (60)$$

$$\phi_m = \frac{s}{\alpha} \quad (61)$$

Now, the stationary DL solution of sG equation can be written as

$$\phi^{(1)} = \frac{\phi_m}{2}\left[1 + \tanh\left(\frac{\xi}{\Delta}\right)\right], \quad (62)$$

where Δ is the width of the DLs, and is given by

$$\Delta = \sqrt{-\frac{24}{\phi_m^2 \alpha}}. \quad (63)$$

VI. NUMERICAL ANALYSIS

It is clear from (62) and (63) that DLs exist if and only if $\mu_0 < 0$. It is obvious from figures 12 to 15 that $\mu > \mu_c$ which confirm us that the DLs are associated with positive potential only. The parametric regimes for the existence of the positive DLs are not bounded by the lower and upper surface plot of μ, and the DLs exist for

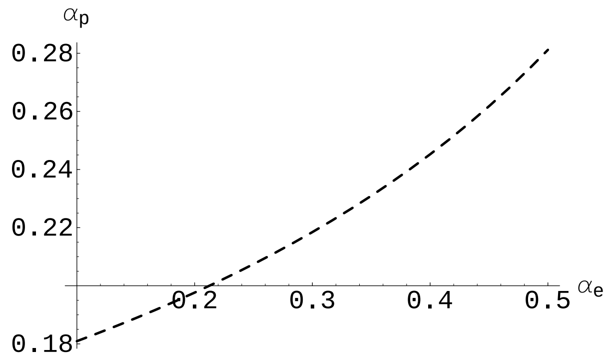

FIG. 1: Showing the 2D graph for the relation between α_e and α_p.

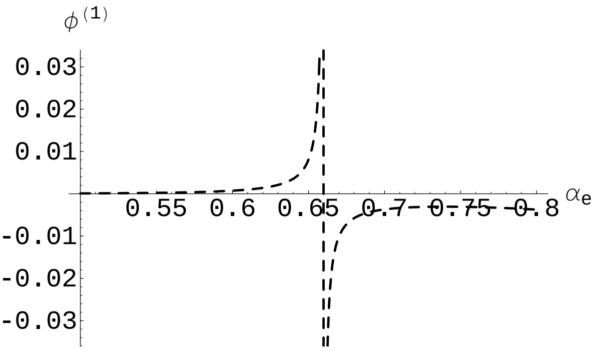

FIG. 2: Showing the 2D graph for the critical value of α_e with respect to $\phi^{(1)}$.

parameters corresponding to any point in between two ($\mu_0 = 0$) surface plots. It may be noted here that if we would neglect the higher order nonlinear term [viz. the third term of Gardner equation or the term containing $\phi^{(3)}$], but would keep the lower order nonlinear term [viz. the second term of Gardner equation or the term containing $\phi^{(2)}$], we would obtain the solitary structures that are due to the balance between nonlinearity (associated with $\phi^{(2)}$ only) and dispersion [58]. On the other hand, in our present work, we have kept both the terms containing $\phi^{(2)}$ and $\phi^{(3)}$, and have obtained the DL structures which are formed due to the balance between the nonlinearity (associated with $\phi^{(2)}$ and $\phi^{(3)}$) and dispersion.

It may be added here that the dissipation (which is usually responsible for the formation of the shock-like structures [59, 60]) is not essential for the formation of the SW and DL structures [61, 62]. It should be noted here that in all these figures we have taken the values of α_e and α_p as a fixed value.

From the first figure we have observed a 2D graphical representation. In this figure a clear relation between α_e and α_p has been observed. It is pointed that the value of α_p slowly increases with the increasing value of α_e. And from the second figure we have got the clear critical value α_e in what range we have got the positive negative poten-

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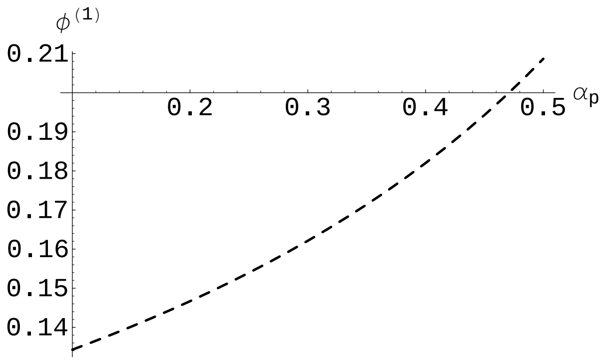

FIG. 3: Showing the 2D graph for α_e with $\phi^{(1)}$.

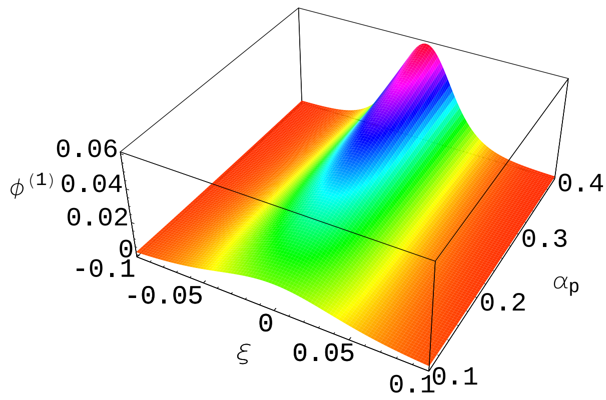

FIG. 6: Showing the effect of μ on SWs (potential structure) for both e-i-p being non-relativistic degenerate when $\alpha_e < 0.66$.

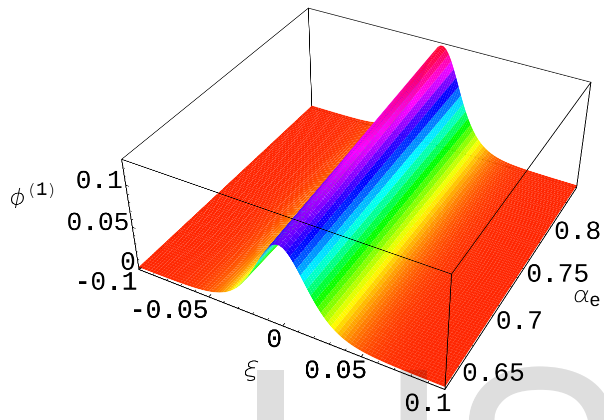

FIG. 4: Showing the effect of α_e on SWs (potential structure) for both e-i-p being non-relativistic degenerate when $\alpha_e < 0.66$.

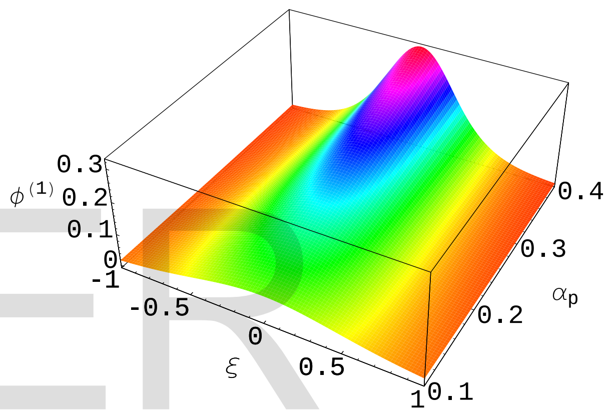

FIG. 7: Showing the effect of α_e on SWs (potential structure) for i being non-relativistic degenerate and e-p being ultra-relativistic degenerate when $\alpha_e < 0.66$.

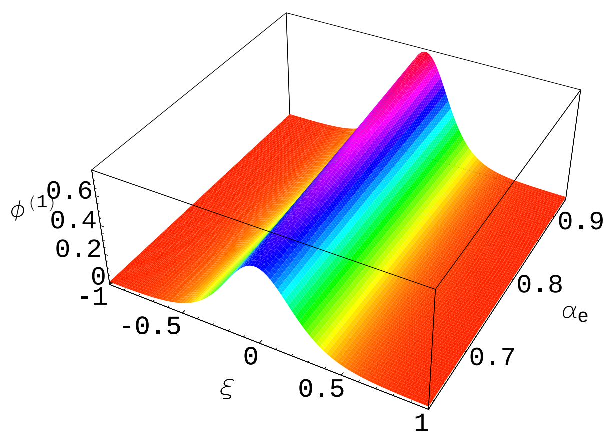

FIG. 5: Showing the effect of α_e on SWs (potential structure) for i being non-relativistic degenerate and e-p being ultra-relativistic degenerate when $\alpha_e < 0.66$.

tial, $\phi^{(1)}$ structures. The third graphical representation confirms us that there is no critical value of α_p for which we may get the positive or negative or both type potential structures; ie; the potential structures do not depend on the value of α_p. In the figures 4-11 we have tried to show the SWs profiles obtained from the stationary solution of SWs for sG equation (55) due to the effect of α_e on the potential, $\phi^{(1)}$ for the case of electron-positron being both non-relativistic and ultra-relativistic degenerate and ion being non-relativistic degenerate. And the figures 12-15 represent the solitons obtainedfrom the stationary solution of DLs for sG equation (62) due to the effect of α_e on the potential, $\phi^{(1)}$ for the both case of relativistic limit. It should be noted here that in all these figures we have taken the values of α_e and u_0 as a fixed value.

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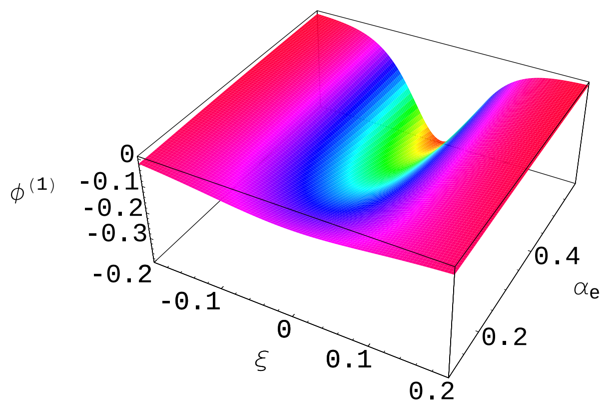

FIG. 8: Showing the effect of α_e on SWs (potential structure) for both e-i-p being non-relativistic degenerate when $\alpha_e \geq 0.66$.

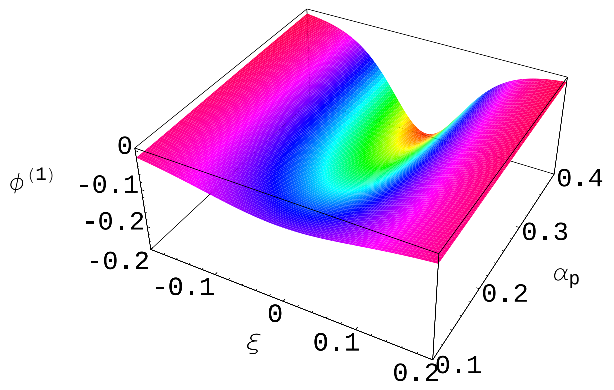

FIG. 10: Showing the effect of μ on SWs (potential structure) for both e-i-p being non-relativistic degenerate when $\alpha_e \geq 0.66$.

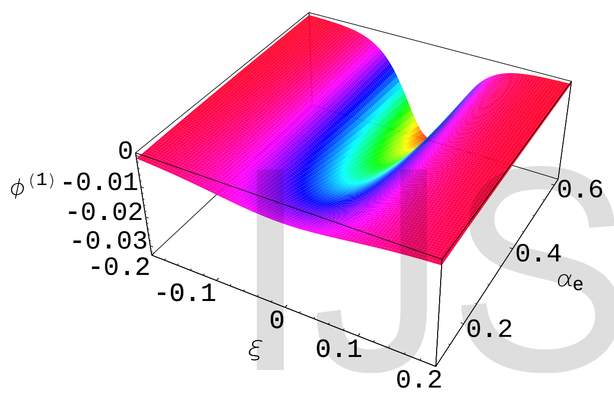

FIG. 9: Showing the effect of α_e on SWs (potential structure) for i being non-relativistic degenerate and e-p being ultra-relativistic degenerate when $\alpha_e \geq 0.66$.

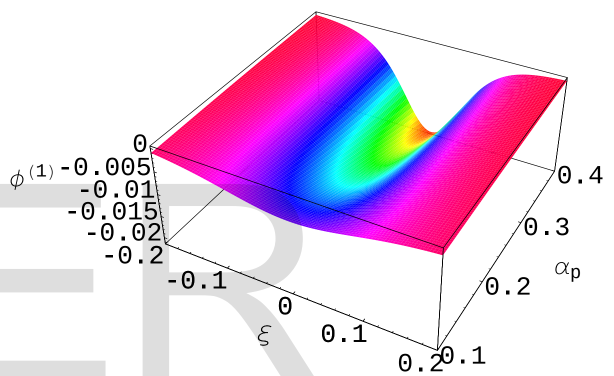

FIG. 11: Showing the effect of α_e on SWs (potential structure) for i being non-relativistic degenerate and e-p being ultra-relativistic degenerate when $\alpha_e \geq 0.66$.

By the careful observation on the figures 4-11 it has become clear that the terms α_e has an great effect on the potential, $\phi^{(1)}$ of SWs which are obtained from the stationary solution of SWs for sG equation (55). Because of the critical value of α_e we get both compressive and rarefactive SWs profiles with the positive and negative potential. Again potential, $\phi^{(1)}$ increases more rapidly for ion being non-relativistic degenerate and electron-positron being ultra-relativistic degenerate than for both electron-positron-ion being non-relativistic degenerate. But we get only positive potential, $\phi^{(1)}$, for the figures of DLs (12-15) for both limits obtained from the solution of standard Gardner equation, whatever the value of α_e, i.e it does not depend on the value of $alpha_e$.

VII. DISCUSSION

We have considered an unmagnetized degenerate dense plasma containing non-relativistic degenerate cold ions fluid and both non-relativistic and ultra-relativistic degenerate electrons and positrons fluid, and have examined the basic features of the electrostatic nonlinear structures that are found to exist in such degenerate dense plasma.

We have investigated the IA SWs and corresponding the DLs in a plasma system (positron fluid, non-relativistic and ultra-relativistic degenerate electrons and non-relativistic degenerate cold ions), by deriving the sG equation. The K-dV solitons and finite amplitude DLs investigated earlier, are not valid for $\alpha_e = (\alpha_e)_c$, which

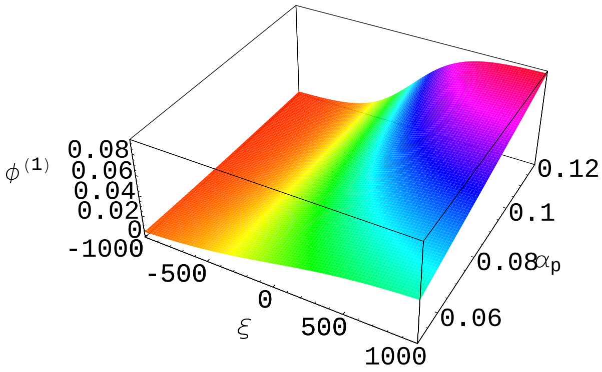

FIG. 12: Showing the effect of α_p on DLs (potential structure) for e-i-p being non-relativistic degenerate when $\alpha_e < 0.66$.

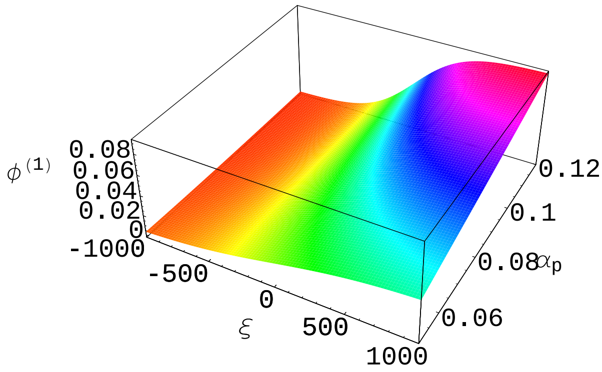

FIG. 14: Showing the effect of α_p on DLs (potential structure) for e-i-p being non-relativistic degenerate when $\alpha_e < 0.66$.

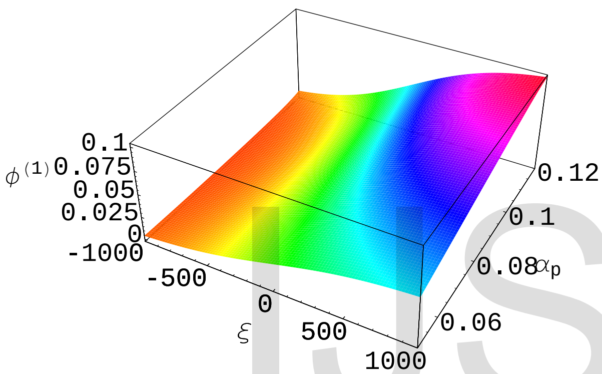

FIG. 13: Showing the effect of α_p on DLs (potential structure) for i being non-relativistic degenerate and e-p being ultra-relativistic degenerate when $\alpha_e < 0.66$.

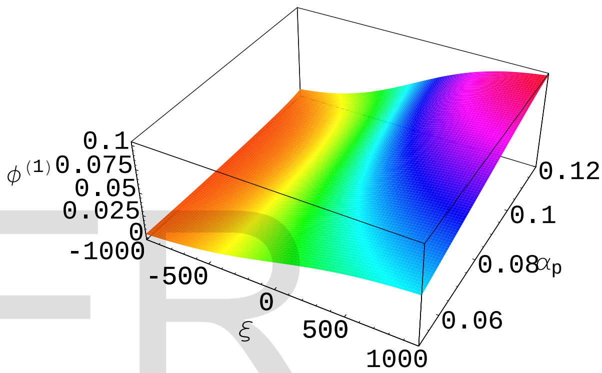

FIG. 15: Showing the effect of α_p on DLs (potential structure) for i being non-relativistic degenerate and e-p being ultra-relativistic degenerate when $\alpha_e < 0.66$.

vanishes the nonlinear coefficients of the K-dV equation.

In short, by observing figures 4-15 it has become clear that
(i) small amplitude SWs with $\phi^{(1)} > 0$, i.e. positive SWs exists if $A > 0$,
(ii) small amplitude SWs with $\phi^{(1)} < 0$, i.e. negative SWs exists if $A < 0$, and
(iii) no SWs can exists around $A = 0$.

(iv) The amplitude and width of SWs increase with μ.

(v) With the increase of the phase speed of plasma species density of ions, the amplitude of SWs does not change significantly.

(vi) The potential of SWs always increases with α_e, but because of critical values of α_e, it changes polarity.

(vii) Only one types of polarity and have no corresponding DLs solution (obtained from (62).

(viii) The width of positive DLs decreases with both α_e and u_0 (nearly negligible).

However, the IA SWs and DLs investigated in our present work are valid for around $\alpha_e = (\alpha_e)_c$. The results, which have been obtained from this investigation can be pinpointed as follows:

1. For such a degenerate plasma system, the potential for the case of ions being non relativistic limits and electron-position being ultra relativistic limits is greater than the electron-positron-ion being non

relativistic limits.

2. The plasma system under consideration supports the finite amplitude SWs and DLs, whose basic features (polarity, amplitude, width, etc.) depend on the degenerate ions and electron-positron-ion number densities.

3. SWs are shown to exist around $\alpha_e = (\alpha_e)_c$, and are found to be different from the K-dV solitons, which do not exist for around $\alpha_e = (\alpha_e)_c$ and mK-dV solitons which exist for around $\alpha_e = (\alpha_e)_c$, but have only one types of polarity and have no corresponding DL solution.

4. At $\alpha_e = (\alpha_e)_c$, negative SWs exist, whereas at $\alpha_e \geq (\alpha_e)_c$, positive SWs exist.

5. The magnitude of the amplitude of positive and negative SWs increases with α_e, but increases with u_0.

6. The DLs having large width we have found only positive potential for both non relativistic and ultra relativistic limits, no negative DLs are formed.

7. The magnitude of the amplitude of the DLs increases with the increase of α_e, also increases with the increase of u_0.

The electrostatic waves in an ultra-relativistic and non-relativistic degenerate dense plasma, which is relevant to interstellar compact objects like white dwarfs, have been investigated. The results, which have been found from this investigation, represent ion acoustic-type of electrostatic waves in which the restoring force comes from the electron-positron degenerate pressure and inertia is provided by the ion mass density. Our studies of nonlinear electrostatic structures in dense e-p-i plasmas with degenerate electrons, ions and positrons are more general. However, arbitrary amplitude IA SWs and DLs in uniform/nonuniform three component degenerate plasma with or without the effects of dust and external magnetic field are also problems of recent interest for many space and laboratory dusty plasma situations, but beyond the scope of our present investigation. Although such plasmas cannot be produced in a laboratory, yet they are gaining considerable attention of the researchers working on dense astrophysical plasmas and numerical simulations.

We have shown the existence of compressive (hump shape) and rarefactive (dip shape) SWs with positive and negative potential and DLs with only positive potential. We have identified the basic features of potential for IA SWs and DLs, which are found to exist beyond the K-dV limit. It may be stressed here that the results of this investigation should be useful for understanding the nonlinear features of electrostatic disturbances in laboratory plasma conditions. Our investigation would also be useful to study the effects of degenerate pressure in interstellar and space plasmas [63], particularly in stellar polytropes [64], hadronic matter and quark-gluon plasma [65], protoneutron stars [66], dark-matter halos [67] etc. The electrostatic waves in an ultra-relativistic and non-relativistic degenerate dense plasma, which is relevant to interstellar compact objects like white dwarfs, have been investigated. The results, which have been found from this investigation, represent ion acoustic-type of electrostatic waves in which the restoring force comes from the electron-positron-ion degenerate pressure and inertia is provided by the ion mass density. We hope that our present investigation will be helpful for understanding the basic features of the localized electrostatic disturbances in compact astrophysical objects (e.g. white dwarfs, neutron stars, black hole, etc.). Further it can be said that the analysis of shock structures, vortices, double-layers etc. in a nonplanar geometry where the degenerate pressure can play the significant role, are also the problems of great importance but beyond the scope of the present work.

To conclude, we propose that a new experiment may be designed based on our results to observe such waves and the effects of planar geometry on these waves in both laboratory and space dusty plasma system. We have carried out SWs and DLs by deriving the standard Gardner equations for a plannar geometry in an unmagnetized plasma system containing degenerate electron-positron (non-relativistic or ultra relativistic limits) and degenerate ions being non-relativistic limit.

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